Design and Analysis of a 2D Truss

MEEN 305-500 – Project 1

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Introduction

In this project, we designed and analyzed a two-dimensional truss structure to determine an optimal layout with optimal cross-sectional geometry for each truss member to carry multiple point loads. To simplify our design process and analysis, we (1) generated truss designs with point loads applied exactly at joint locations, (2) assumed the weight of truss members to be negligible, and (3) assumed “smooth pins” at all truss member joints so that bending moments are neglected. These assumptions allowed us to treat each truss member as two-force members, either in tension or compression. Therefore, the only factors to consider when testing the structural integrity are the yield strength and critical buckling load.

The two-dimensional truss was required to fulfill the following specifications:

- Span a distance of 20 ft.
- Be made of A-36 steel.
- Support a 15-kip load at its center, a 10-kip load located 4 ft from one side, and a 5-kip load 4 ft from the other.
- Contain a minimum safety factor of 1.5 for all truss members under any potential failure mechanism (either yielding or buckling)
- For all truss members to have circular cross-sections

The first part of our design process was to brainstorm potential truss models. Each team member generated at least one unique truss design based on their analysis as seen in **Figures 1-5**. We then determined the minimal amount of mass required to satisfy the design constraints listed above for each truss design, accounting for failure due to either yielding or buckling. The design with the least amount of mass required to satisfy the design requirements was selected as the best model. Finally, we verified our results by calculating the individual factor of safety for each member of our final design, ensuring that they were all at least 1.5.

Truss Model

Individual Models:

As a part of the collaborative brainstorming process, multiple proposals were generated. They intended to take vastly different approaches to the problem to ensure a wide variety of possible solutions were considered. A common analysis process was then generated as a method to effectively determine the best model.

Method 1: Square Frame - Ian Wilhite

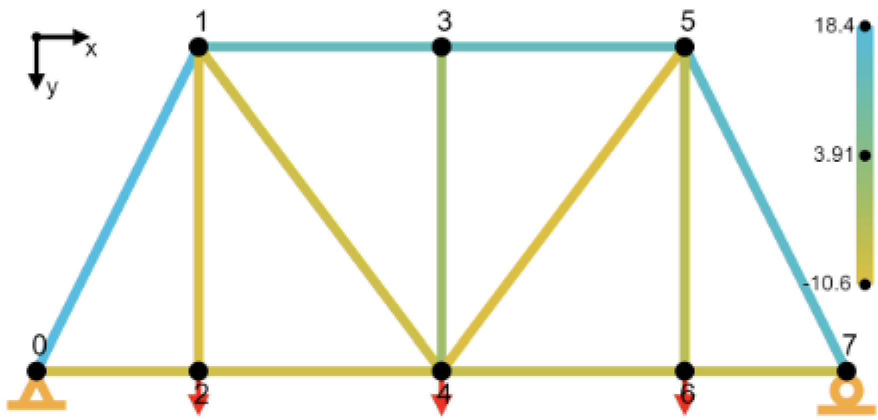


Figure 1. Visual analysis for square frame truss.

This method was intended to take inspiration from many train bridges across the country, and their very square or rectangular appearance by creating parallel beams and adding cross branches to support each segment. The segment widths were modified to allow for loading at the correct increments for analysis.

Method 2: Webbed Frame - David Wood

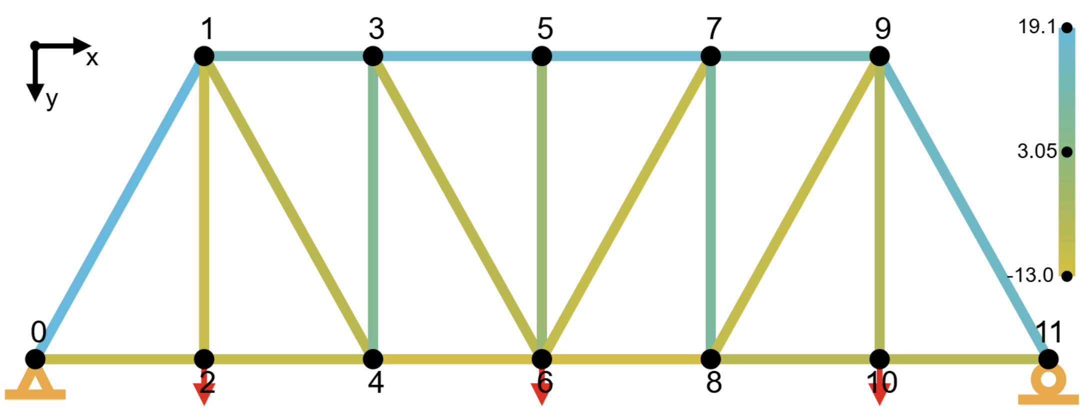


Figure 2. Visual analysis for webbed frame truss.

This model explored whether it would be worth increasing the number of webs in the truss to reduce the overall load for each truss member. While more webs correspond to more needed material, making the bridge heavier, it also contains members significantly shorter in length. This makes them much more resilient to buckling or bending. However, the actual tensile/compressive stress exerted per member remained approximately the same, compared to this same bridge above with 1 web.

Method 3: Parabolic-inspired Frame - Nicholas Licon

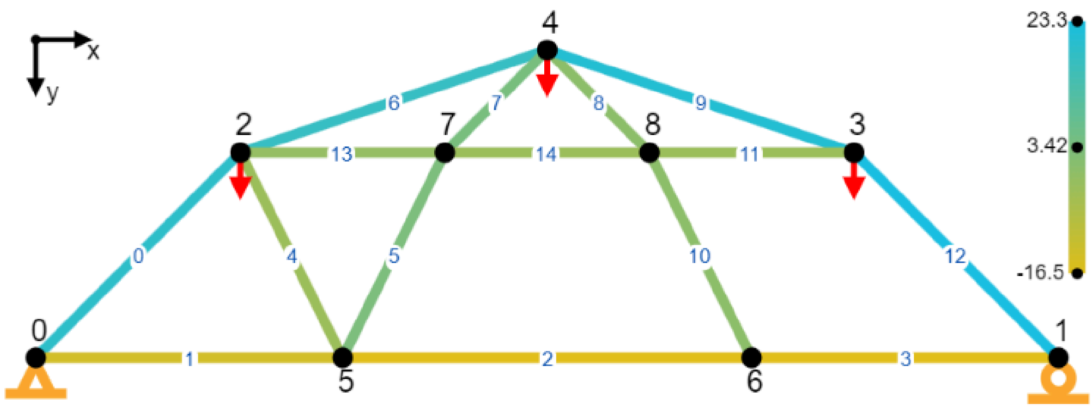


Figure 3. Visual analysis for the Parabolic-inspired Frame.

This shaped truss came from the idea that forces applied to a curved structure are often stronger than others. The shape of a "parabolic" structure distributes the load along the curve of the structure creating members of compression spaced out along the curve as shown with the blue two force members.

Method 4: Pentagonal Frame - Andres Pinzon

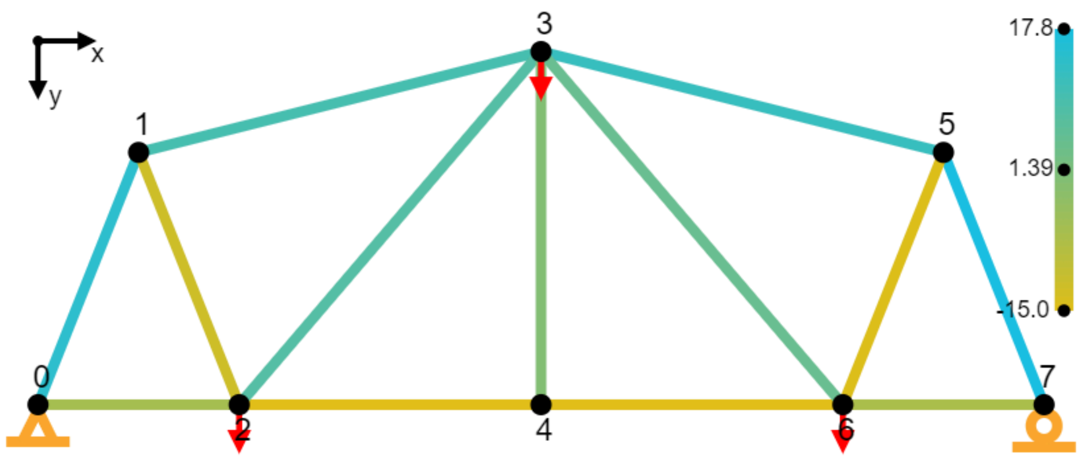


Figure 4. Visual analysis for pentagonal frame truss.

This model was created based on modifications to an already existing warren bridge. The main change is increasing the main member's height. This created a more pentagonal look to the frame and it was noticeably stronger than a simple warren bridge model. The axial force each member sustained was minimized to keep each member's area from growing too much.

Method 5: Triangular Frame - Ian Wilhite

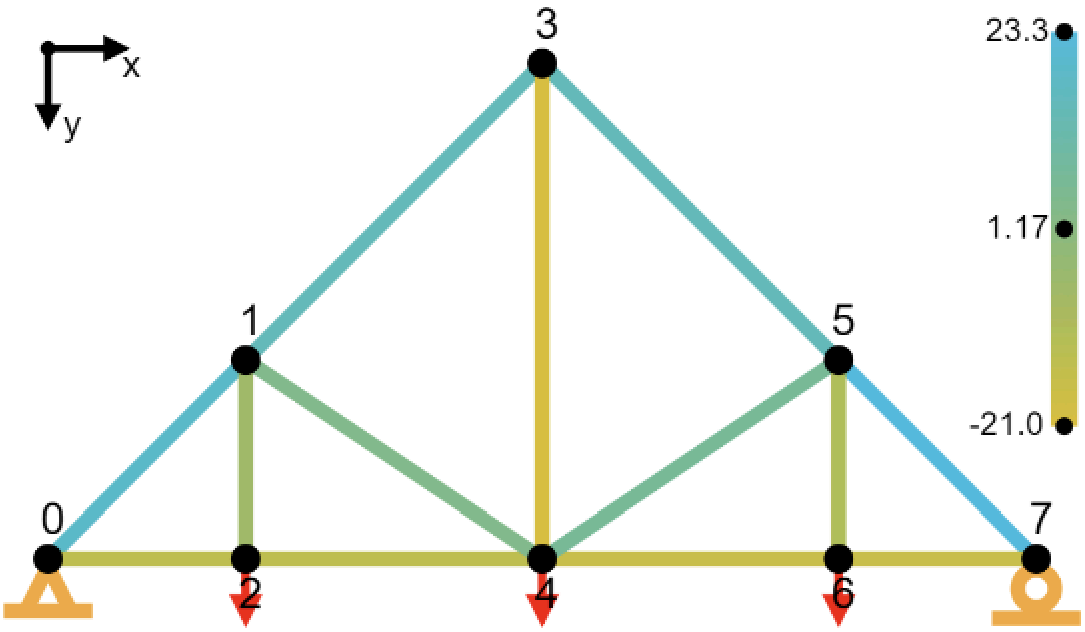


Figure 5. Visual analysis for triangular truss.

This model was meant to remain as simple as possible to theoretically take advantage of the roundup error in frames with many small beams all being rounded up to the nearest quarter inch. This resulted in a substantially weaker truss because the fewer members resulted in less optimal placement of each.

Chosen Model: Parabolic Inspired - Nicholas Licon

This model reduced the total volume of material needed to support the weight. This model leverages a proper balance between linkage placement and roundup radii to generate a truss that can effectively support the loading provided.

Justification of Choice

Each truss was analyzed to determine the total volume of material it would require. The total volume of each truss was then compared to determine the optimal design for the loading conditions.

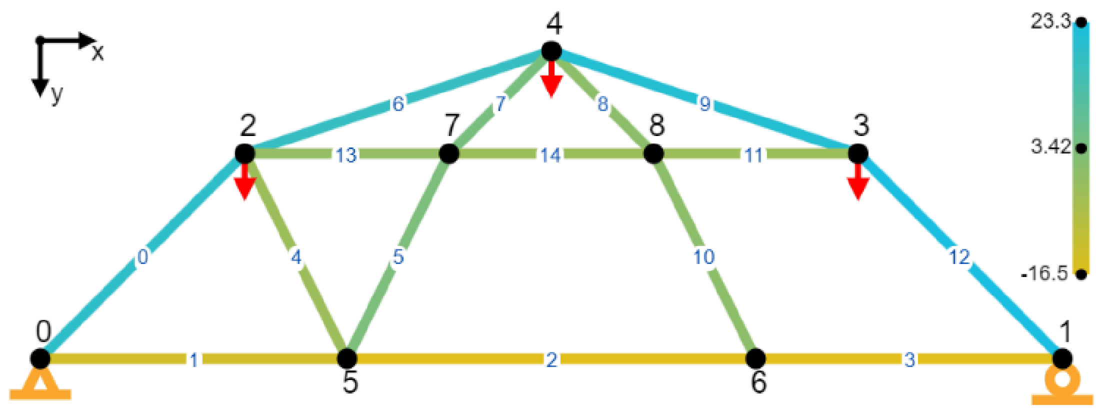
#	Name	Designer	Volume Required (in^3)	Weight Required (lb)
1	Square Frame	Ian Wilhite	2444.240	694.164
2	Webbed Frame	David Wood	1596.073	453.284
3	Parabolic Inspired	Nicholas Licon	1227.537	348.621
4	Pentagonal Frame	Andres Pinzon	2204.261	626.010
5	Triangular Frame	Ian Wilhite	3690.243	1048.029

Table 1. Comparison of bridge designs.

Analysis Process

Each truss presented was analyzed initially using an online calculator which performed the node forces for each truss and presented the forces per member given the loading conditions. These results were imported into a spreadsheet in which each truss could be compared to intake the force, safety factor, and failure stress, to output the total volume and weight of the truss.

Geometric Properties



Density of A-36 steel = 0.284 lb/in^3

RED = Compression	BLUE = Tension

Member	Length (in)	Area (in^2)	Volume (in^3)	Weight (lbs)	FOS
0	67.884	1.767	119.961	34.069	2.555
1	72	0.785	56.545	16.060	2.0943
2	96	0.785	75.398	21.413	1.714
3	72	0.785	56.549	16.060	1.714
4	53.664	0.196	10.537	2.992	2.108
5	53.664	1.227	65.856	18.703	3.551
6	75.9	3.142	238.447	67.719	2.244
7	33.936	0.785	26.653	7.570	2.875
8	33.936	0.0491	1.666	0.473	N/A
9	75.9	3.142	238.447	67.719	1.899
10	53.664	0.0491	2.634	0.748	N/A
11	48	0.196	9.425	2.677	2.356
12	67.884	2.405	163.280	46.372	2.091
13	48	0.196	9.425	2.677	4.712
14	48	0.196	9.425	2.677	2.356

Table 2. Analysis of parabolic inspired truss.

TOTAL VOLUME, TOTAL WEIGHT: **1227.537 in^3**, **348.621 lbs**

Tasks and Responsibilities

Each member was expected to participate in group brainstorming, generate one truss design, and a brief description of their approach and inspiration for their truss. There was also a strong collaborative effort in gaining a fundamental understanding of the equations, simulation tools, and theory behind what constitutes a good truss.

Ian Wilhite - Ian was responsible for creating the spreadsheet to analyze the trusses, the framework of the report, the chosen model, and Tasks and Responsibilities.

Nicholas Licon - Nicholas was responsible for the Introduction.

Andres Pinzon Diaz - Andres was responsible for the geometric properties, excel appendix.

David Wood - David was responsible for the Introduction, and the generation of Geometric Properties for the selected truss.

Appendix

Matlab code for solving forces:

```
% Number of nodes and members
n_nodes = 9;
n_members = 15;

% Node coordinates (x, y)
nodes = [0, 0; 20, 0; 4, 4; 16, 4; 10, 6; 6, 0; 14, 0; 8, 4; 12, 4];
% External forces (Fx, Fy) at each node
forces = [0, 0; 0, 0; 0, -5; 0, -10; 0, -15; 0, 0; 0, 0; 0, 0; 0, 0];
% Connectivity matrix (start node, end node)
members = [0, 2; 0, 5; 5, 6; 6, 1; 2, 5; 7, 5; 2, 4; 4, 7; 4, 8; 4, 3; 6, 8; 3,
8; 3, 1; 2, 7; 7, 8] + 1;

% Initialize system of equations
n_eqs = 2 * n_nodes; % 2 equations (Fx and Fy) per node
A = zeros(n_eqs, n_members + 3); % Matrix of coefficients
b = zeros(n_eqs, 1);             % Vector of constants
% Reaction forces (supports at nodes 0 and 1)
% Loop through each member and define equations based on equilibrium
for member_id = 1:n_members
   n1 = members(member_id, 1);
   n2 = members(member_id, 2);

   % Calculate direction cosines (l, m)
   dx = nodes(n2, 1) - nodes(n1, 1);
   dy = nodes(n2, 2) - nodes(n1, 2);
   L = sqrt(dx^2 + dy^2);
   l = dx / L;
   m = dy / L;

   % For node n1 (Fx and Fy)
   A(2*n1-1, member_id) = l;  % Fx equation for node n1
   A(2*n1, member_id) = m;    % Fy equation for node n1
```

```matlab
    % For node n2 (Fx and Fy)
    A(2*n2-1, member_id) = -l; % Fx equation for node n2
    A(2*n2, member_id) = -m;   % Fy equation for node n2
end
A(1, n_members+1) = 1; A(2, n_members+2) = 1;
A(4, n_members+3) = 1;
for i = 1:n_nodes
    b(2*i-1) = forces(i, 1); % Fx
    b(2*i) = forces(i, 2);   % Fy
end
% Solve for forces in each member and support reactions
disp(A);
disp(b);
x = A \ b;
% Display the results
forces_in_members = x(1:n_members);
reaction_forces = x(n_members+1:end);
disp('Forces in members (kips):');
disp(forces_in_members);
disp('Reaction forces (kips):');
disp(reaction_forces);
```

Output of MatLab code:

```
Forces in members (kips):
  19.0919
 -13.5000
 -16.5000
 -16.5000
  -3.3541
   3.3541
  17.3925
   4.2426
   0.0000
  20.5548
   0.0000
  -3.0000
  23.3345
  -1.5000
  -3.0000

Reaction forces (kips):
   0.0000
 -13.5000
 -16.5000
```

Truss visualization:

trussanalysis.com

Excel sheets used to calculate geometric properties:

2		**Young's Modulu**	**A-36 Yield Strength**		**Factor of Safety**				
3		**29000**	**36000**		**1.5**				
4			36						
5									
6	A	B	C	D	E	F	H	I	J
7	**Member ID**	**Start -> End Nod**	**Length (ft)**	**Axial Force (kips)**	**force * FoS**	**Area (in^2)**	**Radius - yield (in)**	**Moment of Inertia, I (lb**	**Radius - gyration (in)**
8	0	0 → 2	5.657	19.09	28.635	0.7954166667	0.50	0.79	0.88
9	1	0 → 5	6	-13.5	-20.25	0.5625	0.42	0.05	0.00
10	2	5 → 6	8	-16.5	-24.75	0.6875	0.47	0.05	0.00
11	3	6 → 1	6	-16.5	-24.75	0.6875	0.47	0.05	0.00
12	4	2 → 5	4.472	-3.354	-5.031	0.13975	0.21	0.00	0.00
13	5	7 → 5	4.472	3.354	5.031	0.13975	0.21	0.12	0.50
14	6	2 → 4	6.325	17.39	26.085	0.7245833333	0.48	0.79	0.90
15	7	4 → 7	2.828	4.243	6.3645	0.1767916667	0.24	0.05	0.42
16	8	4 → 8	2.828	0	0	0	0.00	0.00	0.00
17	9	4 → 3	6.325	20.55	30.825	0.85625	0.52	0.79	0.94
18	10	6 → 8	4.472	0	0	0	0.00	0.00	0.00
19	11	3 → 8	4	-3	-4.5	0.125	0.20	0.00	0.00
20	12	3 → 1	5.657	23.33	34.995	0.9720833333	0.56	0.79	0.92
21	13	2 → 7	4	-1.5	-2.25	0.0625	0.14	0.00	0.00
22	14	7 → 8	4	-3	-4.5	0.125	0.20	0.00	0.00

2				**Total Vol (in ^3)**	**Total Mass (lb)**				
3				**1227.537**	**348.621**				
4									
5									
6	K	L	M	N	O	P	Q	R	X
7	**Radius - Failure (**	**Radius to 1/8th (in)**	**Area(in^2)**	**Volume (in^3)**	**Weight (lb)**	**FOS t**	Pcr	FOS c	**FOS Consoldiated**
8	0.88	1.00	3.14	213.26	60.57	5.92	48.78	2.56	**2.56**
9	0.42	0.50	0.79	56.55	16.06	-2.09	2.71	0.00	**2.09**
10	0.47	0.50	0.79	75.40	21.41	-1.71	1.52	0.00	**1.71**
11	0.47	0.50	0.79	56.55	16.06	-1.71	2.71	0.00	**1.71**
12	0.21	0.25	0.20	10.54	2.99	-2.11	0.30	0.00	**2.11**
13	0.50	0.63	1.23	65.86	18.70	13.17	11.91	3.55	**3.55**
14	0.90	1.00	3.14	238.45	67.72	6.50	39.02	2.24	**2.24**
15	0.42	0.50	0.79	26.65	7.57	6.66	12.20	2.88	**2.88**
16	0.00	0.13	0.05	1.67	0.47	N/A	0.05	0.00	**0.00**
17	0.94	1.00	3.14	238.45	67.72	5.50	39.02	1.90	**1.90**
18	0.00	0.13	0.05	2.63	0.75	N/A	0.02	0.00	**0.00**
19	0.20	0.25	0.20	9.42	2.68	-2.36	0.38	0.00	**2.36**
20	0.92	1.00	3.14	213.26	60.57	4.85	48.78	2.09	**2.09**
21	0.14	0.25	0.20	9.42	2.68	-4.71	0.38	0.00	**4.71**
22	0.20	0.25	0.20	9.42	2.68	-2.36	0.38	0.00	**2.36**

<u>Equations for Excel Columns:</u>

Column	***A***	***B***	***C***
Equation	*Member ID*	*Member Start->End*	*Length*

Column	***D***	***E***	***F***
Equation	*Axial Force (kips)*	`=D8*$E$3`	`=abs(E8*1000/$C$3)`

Column	***H***	***I***	***J***
Equation	`=sqrt(F8 / PI())`	`=PI()/4*L8^4`	`=if(D8>0,(E8*1000*C8*C8*12^2/(PI()^3*$B$3*1000*0.25))^(1/4),0)`

Column	***K***	***L***	***M***
Equation	`=max(J8,H8)`	`=floor(K8, 1/8) + 0.125`	`=pi()*L8^2`

Column	***N***	***O***	***P***
Equation	`=C8*pi()*L8^2*12`	`=N8*0.284`	`=$C$4/(D8/M8)`

Column	***Q***	***R***	***X***
Equation	`=(PI()^2 * 29000*I8) / (C8*12)^2`	`=if(D8>0,Q8/D8,0)`	`=if(P8<0,-P8,R8)`