Meen 305 Project 2: Flower Petal Loadings

By David Wood, Nick Licon, Andres Pinzon, and Ian Wilhite

Section 1: Problem Summary

The client has a flower solar panel of 8 blades. The task provided was to design a truss and column structure that could support the mass of the blades and their loading during usage. The objective of this design is to minimize the mass of the column and truss with a given deflection while keeping reasonable factors of safety.

Section 2: Design Process

The column must not fail due to yielding/fracturing due to stresses, buckling due to applied load, or laterally deflect more than 0.3 in.

To get a radius that would create a maximum deflection of 0.3 in, Equations () were used. Then, the maximum stresses at critical points were calculated on either side of the column to make sure the column would not yield under the load. Additionally, the critical buckling load was calculated so that we could ensure the column would not buckle, since oftentimes a column will buckle before it yields under a compressive load.

There were a number of design constraints applied to ensure valid :

- Lateral deflection not to exceed 0.3 inches.
- Factor of safety on all calculated values of at least 2.
- Constant thickness of the column vertically (as opposed to varied thickness as a function of height) for ease of manufacturing and ability to use existing commercial products.
- Column may be hollow to allow for more efficient use of material.
- All column measurements must be in increments of ¼ inch for ease of manufacturing.

Failure due to Yielding:

The first thing we noticed for the column was the column would experience maximal stress when the solar panels were in the vertical (90°) position. This is because the sum of the panel components acts as a concentrated load 1 ft away from the column's central axis, generating a bending stress. Furthermore, the wind force, which is also modeled as concentrated load, is also highest when the panels are in the vertical position. If the wind force acts in the direction so that it hits the back of the panels, it will generate an additional bending moment and stress in the same direction. To illustrate this, three free body diagrams with different configurations are provided below to display our thought process and logically lead to the highest failure scenario for the column.

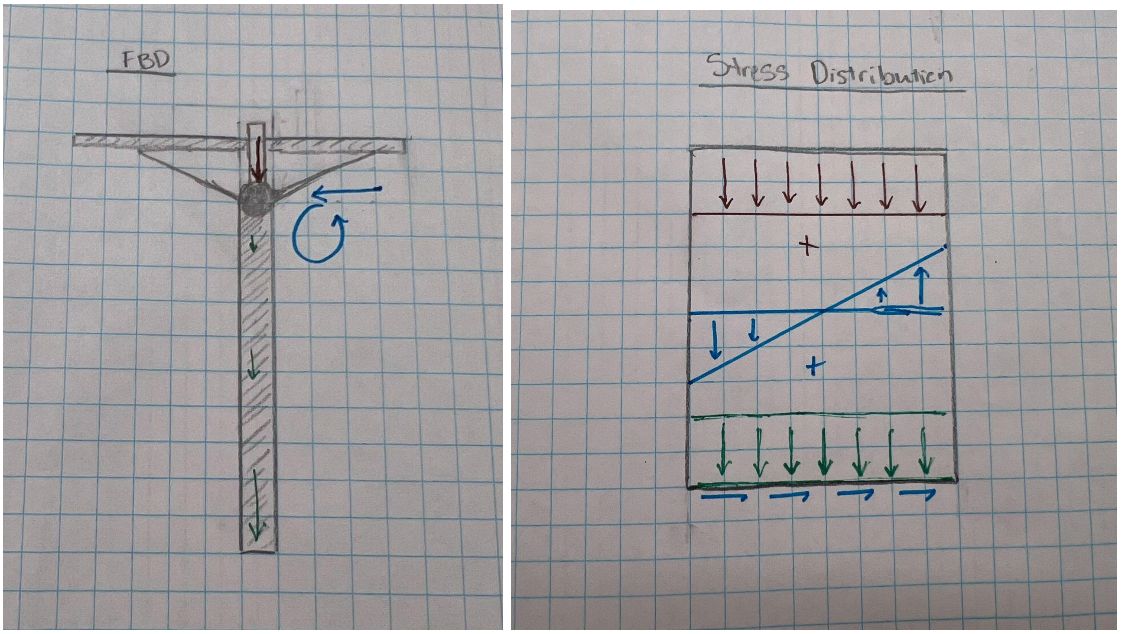


Figure 1. Force/Moment Diagram and Stress Distribution for Flat Position

In this configuration, there is no bending stress generated by the solar panels. Furthermore, the bending stress generated by the wind is smaller because the wind force data is provided and approximated at a lower value of 40 lbs. The column should not be designed to avoid failure under this scenario because there are other configurations that can generate much higher stresses. The same logic applied for the angled (45°) position. There is some bending stress generated by the weight of the solar panels, and the bending stress generated by the wind is also higher too (wind force modeled at 85 lbs), but they are both not as great as the vertical position.

A moment is calculated by multiplying the magnitude of a force and its perpendicular distance from an axis by which it revolves. In the case of the bending stress generated by the solar panels, the *radius* is maximal in the vertical position. For the bending stress generated by the wind, the *force* is maximal in the vertical position. Therefore, we decided to not design the column based on an angled configuration because, logically, there are other configurations that generate higher stresses.

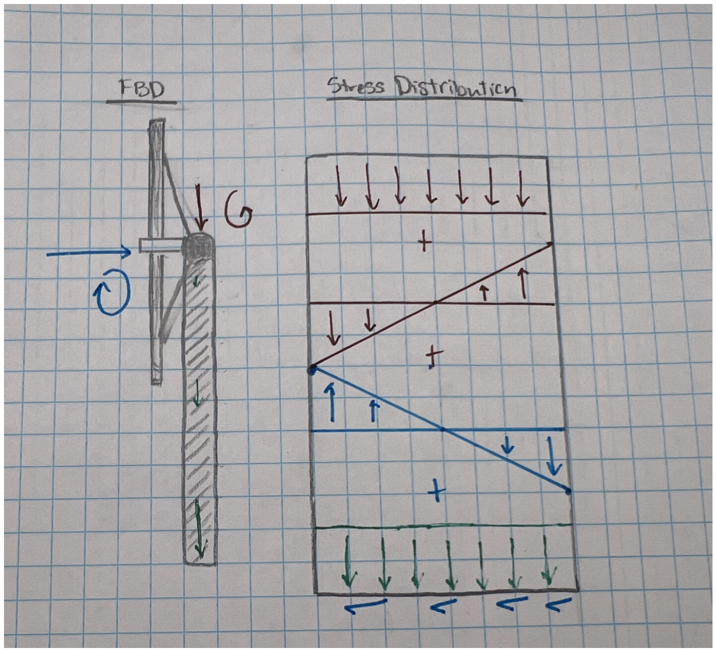


Figure 2. Force/Moment Diagram and Stress Distribution for Oncoming Wind

In this configuration, the bending moments generated by the weight of the solar panels and the wind partially offset each other because they are acting opposite in direction. As seen in **Figure 2**, on the left side of the column, the solar panels generate a maximum compressive stress, while the wind force generates a maximum tensile stress. On the right side of the column, the solar panels generate a tensile stress and the wind force generates a compressive stress. Regardless of where, the bending stresses are always cancelling each other out to some degree.

The last consideration is where the force is applied on the back side of the panels. This would generate much higher stresses because the bending moments would be acting in the same direction, with maximum compression on the left side and maximum tension on the right, as you can see below:

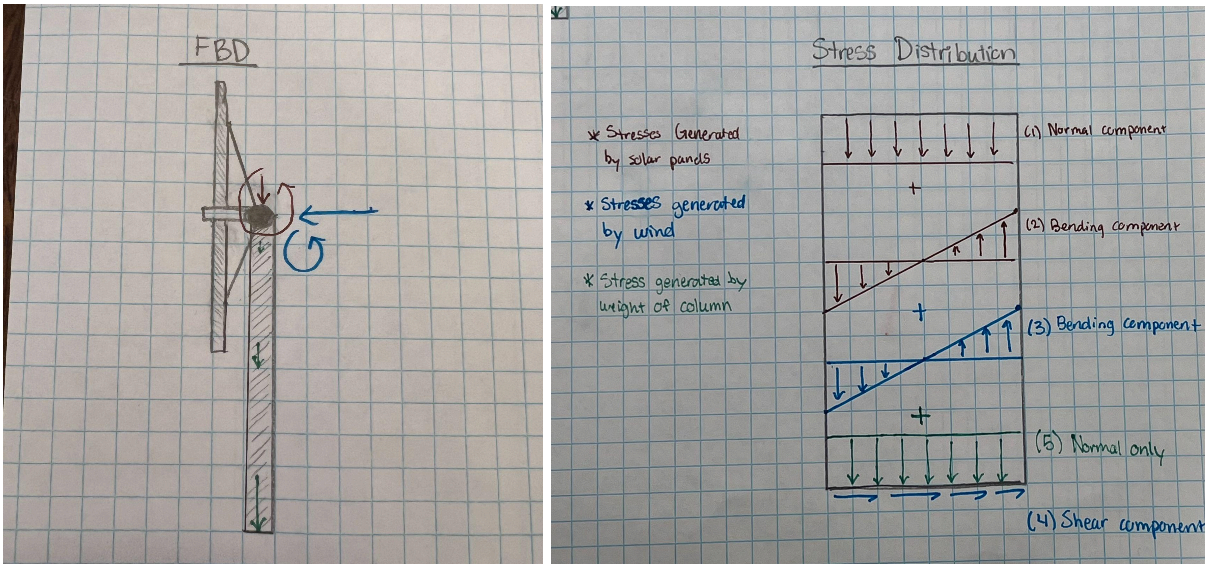


Figure 3. Force/Moment Diagram and Stress Distribution of Highest Failure Scenario

This is the configuration for the column by which the most stress is generated, and the one we need to design around. As you can see, the weight of the solar panels generates a stress that has both normal and bending components, while the wind force generates a stress that has shear and bending components. The weight of the column itself generates stress too, but unlike the other stresses, it varies with the length of the column. The normal stress due to the weight of the column is maximal at the bottom and zero at the top of the column.

The normal stress components have a uniform distribution, meaning that the magnitude and direction of the stress is constant at any individual location within the column's cross-section. Bending stresses, on the other hand, have a linear distribution based on the distance it is from the central axis of the column. The magnitude of this stress is the same in either direction from the center but opposite in direction, depending on whether you are closer or further away from where the moment-inducing force is being applied. On the left side, it generates a maximum compressive stress, while on the right it generates a maximum tensile stress.

The formulas used to calculate all stresses exerted on the column are summarized below:

Normal Stress	Shear Stress	Bending Stress	Torsional Stress
$\sigma = \frac{P}{A}$	$\tau = \frac{VQ}{It}$	$\sigma = -\frac{My}{I}$	$\tau = \frac{T\rho}{J}$

Brittle Materials

Brittle materials have different strengths in the tensile and compressive directions, being much stronger in compression than tension. This rules out our ability to use the **maximum normal stress theory**, which is derived based on the assumption that the material is equally strong in compressive/tensile directions. Instead, we will find the locations in the column where stress is maximum in either compressive or tensile directions, and compare them to the compressive/tensile stresses provided for each material.

We determined that there are two locations on the beam where failure is most likely to occur. Referencing the two-dimensional stress distribution layout in **Figure 3**, these points would be the lower left-hand corner and the upper-right hand corner. As the lower left location, the column is under **maximal compressive stress.** The compressive stresses add upon each other, and, due to the nature of each distribution, they accumulate to a maximum at this location. In the upper right location, the column is potentially under **maximal tensile stress**.

NOTE: The word "potentially" is used because the downward normal components acting on the right side of the column may be greater in magnitude than the upward components due to bending, but this is highly unlikely.

Ductile Materials

There is no data given on compressive/tensile strengths for the ductile materials. This is likely because yielding in ductile materials is dominated by shear mechanisms, and they are not super direction-dependent. Therefore, we will assume that the ductile materials are equally strong in compressive/tensile directions and use the **maximum shear stress theory** (or Tresca criterion) to determine if it will yield under compressive loads. Note that the magnitude of maximum compression at a location will be greater than the maximum tension at another. The yield strength is estimated based on the formula:

$$\tau_{\substack{abs \\ max}} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_y}{2}$$

The shear stress generated by the wind is neglected because it is relatively small (see **Appendix B.3**), so there is no need to determine the principal stresses. The maximum stress will simply be the lower-left corner (see **Figure 3)** where it is under maximum compression.

Failure Due to Buckling

Buckling also had to be taken into consideration as the weight of the column could cause it to fail on itself. This occurs when the forces acting on the column overcome its ability to resist bending, and this depends on its stiffness (bending rigidity, EI), how it's supported, and its length. In this case, with a fixed-free setup, the column's weight itself can create enough stress to make it buckle when it reaches a critical value. It's like a battle between the column's rigidity and the load trying to push it into instability.

$$W_c = 7.84 \frac{EI}{L^2}$$

This critical weight required to maintain the integrity of the column did not make much of an impact in the end. Since we were minimizing the weight of the column, our results yielded values that were much lower. Nonetheless, these calculations are important to make as they ensure this specific part of the design is secure and it adds an extra layer of credibility to our final design.

Minimizing Lateral Deflection to less than 0.3 inches

To make sure the column did not deflect more than 0.3 inches there were two scenarios that had to be considered. First we were tasked with finding the most suitable design without any wind forces, so the only thing causing the column to deflect was the bending moment caused by the blades.

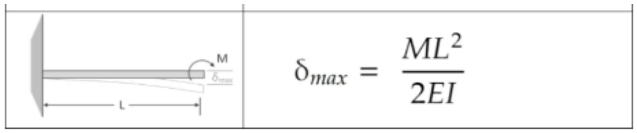


Figure 4. Deflection due to bending stress

Due to this bending effect, the lateral deflection of the column was found using the equation seen in **Figure 4**. A design was then optimized to have a FOS that was less than 2.

Once wind was taken into consideration, another deflection was required to be calculated. This time the column was going to be deflected due to a force that pushes sideways and would make the column bow out over its length.

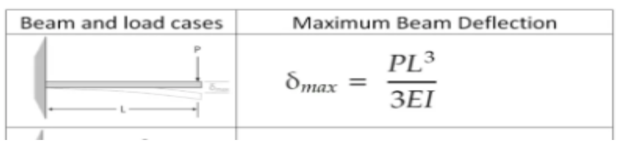


Figure 5. Deflection due to Force

With a new input causing a deflection on the column by the amount seen in the equation in **Figure 5**, our FOS changed from our minimum of 2 down to less than 1 for our chosen model. This would mean that the column would deflect more than the expected 3 inches. A new inner and outer radius were constructed to make sure that the sum of the deflections caused by both the wind and the bending would not exceed 3 inches.

Column Conclusions

For the column, it was clear that limiting the deflection to 0.3 inches was the primary limiting factor for design optimization. Calculations of failure due to buckling or yielding/fracturing did not even come close to their determined limits.

Section 2.2 Design Considerations for Truss:

The Truss must not fail due to buckling or normal stress under the load of the panel and must pass with a factor of safety greater than 2. The analysis of the truss problem is a statically indeterminate problem. Connected to a rigid wall there are 3 unknowns due to the cantilever beam-like properties it will have. Additionally, the force of the truss acts upward three feet from the wall. This means that we must use analysis of beam deflection to determine the fourth equation to solve for all four unknowns. To solve the beam deflection problem we will assume the compatibility to be no deflection at the end of the panel meaning that at what the truss is supporting the weight the displacement is 0 (L=3, v = 0).

Once the force acting on the truss is found a test for both brittle materials and ductile materials will be performed as well as for what the radius of the cylindrical solid truss should be.

Position one of the flowers allows us to ignore the truss in that position due to the compatibility of the orientation and the bar. With the weight going straight down the only compatibility for the panel that we know is that the displacement of the bar in the x direction is zero, meaning that the trust can exert any force upon the panel in that position leading no force to be placed on the truss in this position.

Volume = Area * length
Density(lbs/in^3) = Density(lbs/ft^3) / (12^3)

Material	Minimum radius	Volume	Weight
Gray cast iron 30	0.220in	5.7699in^3	1.402lbs
Gray cast iron 60	0.205in	5.01in^3	1.450lbs
Stainless steel 304	0.189in	4.258in^3	0.4189lbs
Aluminum Alloy	0.243in	7.0395in^3	0.387lbs

Table 1. Evaluation of Truss

Aluminum Alloy gives the same amount of support as the others for the minimized weight of the truss making it the best material to use for our given case.

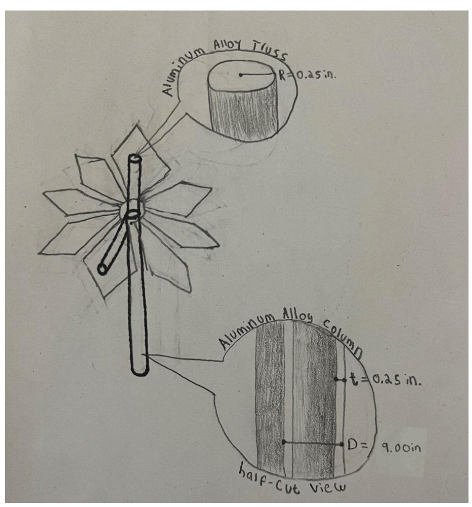


Figure 6. Sketch of Truss6

Section 3: Results

Section 3.1 Calculated Values

Parameter	Value
Maximum Lateral Deflection	0.3 [in]
Flower Weight	40 [lbs]
Column Height	10 [ft]
Number of Flowers	8
Minimum Factor of Safety	2
End condition factor	0.25

Table 2. Assumed parameters

Material	Material Type	Inner Radius (in)	Outer Radius (in)	Density (lb/ft^3)	Mass (lb)	FOS
Cast Iron 30	Brittle	3.75	4	420	177.5	2.127
Cast Iron 60	Brittle	3.5	3.75	500	197.7	2.322
Stainless steel 304	Ductile	3.00	3.25	170	58.0	2.047
Aluminum Alloy	Ductile	4.25	4.50	95	45.3	2.071

Table 3. Evaluation of Column

Section 3.1 Final Recommendations

Section	Material	Mass (lb)
Truss	Aluminum	0.387
Column	Aluminum	45.3
Total		45.7

Table 4. Chosen Masses

To minimize weight, both parts should be made of the aluminum alloy option, and the total structure will have a total weight of 45.7 lbs.

Conclusion

For the column, it was clear that limiting the deflection was the primary limiting factor for design optimization. Calculations of failure due to buckling or yielding/fracturing did not even come close to their determined limits. It could be inferred that in future analysis, this limit could be revisited as a design constraint. For the truss, the main limiting design factor was critical buckling load. The designs proposed meet the specified design constraints, and would perform as expected if constructed. Next steps for the project could include verification of calculated values via simulation, or scale tests to ensure the assumptions provided hold true. The design implemented effectively minimizes the weight of the material of the structure, minimizing the error from excluding their weight while calculating stresses.

Section 4: Member Contributions

The following is a summary of the contributions each member provided to the generation of the design and report presented:

- David Wood: David worked on developing the calculator for the column, specifically for the brittle materials. He also evaluated the columns compressive and tensile strength, and took the lead on debugging the calculator.
- Nick Licon: Nick developed the calculator for the truss for both ductile and brittle materials. He also introduced many of the analysis techniques that were crucial to evaluating the column.
- Andres Pinzon Diaz: Andres primarily worked on understanding the column buckling equations and their applications, then their implementation in the column.
- Ian Wilhite: Ian worked on developing the calculator for the column, specifically for the ductile materials. He also worked on preliminary reporting and approach structure.

Appendix

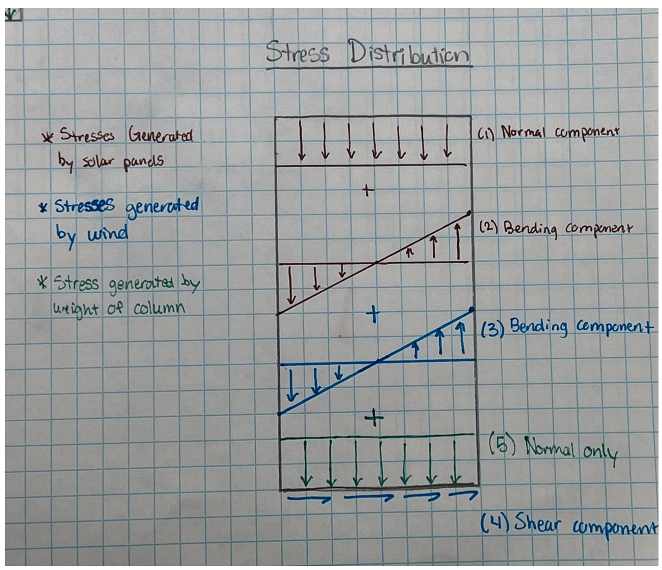
A. Tables

Table 1 Material Properties

Material	Density (lb/ft^3)	Elastic Modulus (ksi)	Tensile Yield Stress (ksi)	Tensile Ultimate Stress (ksi)	Compressive Ultimate Stress (ksi)	Shear Yield or Ultimate Stress (ksi)
Gray cast iron 30	420	15000		30	100	15
Gray cast iron 60	500	20000		60	180	30
Stainless steel 304	170	27500	40			20
Aluminum Alloy	95	10150	43			20

B. Calculations

<u>B.1 Column Stress Analysis and Dimensional Optimization of Brittle Material Options</u>



- Max compressive stress (lower left corner) = $\sigma_{panels,\ normal} + \sigma_{panels,\ bending} + \sigma_{wind,\ bending} + \sigma_{column\ weight}$
- Max Tensile Stress (upper right corner) = $\sigma_{panels,\ normal} + \sigma_{panels,\ bending} + \sigma_{wind,\ bending}$

SIGN CONVENTION: Negative sign is used for compression; positive sign is used for tension. So stresses must be <u>greater than</u> the maximum (negative) compressive stress, and <u>less than</u> the maximum (positive) tensile stress

$$\sigma_{max,\ compression} = -\frac{(320\ lbs)}{\pi\left(r_o^2 - r_i^2\right)} - \frac{(320\ ft\cdot lbs)\cdot r_o}{\frac{\pi}{4}\left(r_o^4 - r_i^4\right)} - \frac{(120lbs\cdot 120in)\cdot r_0}{\frac{\pi}{4}\left(r_o^4 - r_i^4\right)} - \frac{\rho\cdot\pi\left(r_o^2 - r_i^2\right)(120in)\cdot g}{\pi\left(r_o^2 - r_i^2\right)}$$

$$\sigma_{max,\ tension} = -\frac{(320\ lbs)}{\pi\left(r_o^2 - r_i^2\right)} + \frac{(320\ ft\cdot lbs)\cdot r_o}{\frac{\pi}{4}\left(r_o^4 - r_i^4\right)} + \frac{(120lbs\cdot 120in)\cdot r_0}{\frac{\pi}{4}\left(r_o^4 - r_i^4\right)}$$

<u>Gray Cast Iron 30:</u> r_0 = 4.0 in, r_i = 3.75 in

$$\sigma_{max,\ compression} = -\frac{(320\ lbs)}{\pi\left(4^2 - 3.75^2\right)} - \frac{(320\ ft\cdot lbs)\cdot 4}{\frac{\pi}{4}\left(4^4 - 3.75^4\right)} - \frac{(120lbs\cdot 120in)\cdot 4}{\frac{\pi}{4}\left(4^4 - 3.75^4\right)} - \frac{0.243\cdot\pi\left(4^2 - 3.75^2\right)(120in)\cdot 32.2}{\pi\left(4^2 - 3.75^2\right)}$$

= **-2.586619605 ksi > -100 ksi;** well within failure limit

**The density was converted to lbs/in^3*

$$\sigma_{max,\, tension} = -\frac{(320\ lbs)}{\pi\left(4^2-3.75^2\right)} + \frac{(320\ ft\cdot lbs)\cdot 4}{\frac{\pi}{4}\left(4^4-3.75^4\right)} + \frac{(120lbs\cdot 120in)\cdot 4}{\frac{\pi}{4}\left(4^4-3.75^4\right)}$$

= **1.542307995 ksi < 30 ksi;** well within failure limit

Gray Cast Iron 60: r_0 = 3.75in, r_i = 3.50 in

$$\sigma_{max,\, compression} = -\frac{(320\ lbs)}{\pi\left(3.75^2-3.5^2\right)} - \frac{(320\ ft\cdot lbs)\cdot 3.75}{\frac{\pi}{4}\left(3.75^4-3.5^4\right)} - \frac{(120lbs\cdot 120in)\cdot 3.75}{\frac{\pi}{4}\left(3.75^4-3.5^4\right)} - \frac{0.289\cdot\pi\left(3.75^2-3.5^2\right)(120in)\cdot 32.2}{\pi\left(3.75^2-3.5^2\right)}$$

= **-3.000360171 ksi > -180 ksi;** well within failure limit

$$\sigma_{max,\, tension} = -\frac{(320\ lbs)}{\pi\left(4^2-3.75^2\right)} + \frac{(320\ ft\cdot lbs)\cdot 4}{\frac{\pi}{4}\left(4^4-3.75^4\right)} + \frac{(120lbs\cdot 120in)\cdot 4}{\frac{\pi}{4}\left(4^4-3.75^4\right)}$$

= **1.769908297 ksi < 60 ksi;** well within failure limit

B.2 Column Stress Analysis and Dimensional Optimization of Ductile Material Options

- Stress, is any direction, must be less than $\sigma_{yield} = 2^*\tau_{ultimate}$, per Tresca criterion. σ_{yield} = 2 * 20ksi (for both ductile materials) = **40 ksi.**
- The maximum stress in compression will always be greater than the maximum stress in tension, so there is no need to calculate maximum tension.

Stainless Steel 304: r_0 = 3.25 in, r_i = 3.0 in

$$\sigma_{max,\, compression} = -\frac{(320\ lbs)}{\pi\left(3.25^2-3^2\right)} - \frac{(320\ ft\cdot lbs)\cdot 3.25}{\frac{\pi}{4}\left(3.25^4-3^4\right)} - \frac{(120lbs\cdot 120in)\cdot 3.25}{\frac{\pi}{4}\left(3.25^4-3^4\right)} - \frac{0.0984\cdot\pi\left(3.25^2-3^2\right)(120in)\cdot 32.2}{\pi\left(3.25^2-3^2\right)}$$

= **-2.914629187 ksi > -40 ksi;** well within failure limit

**The density was converted to lbs/in^3*

Aluminum Alloy: r_0 = 4.5 in, r_i = 4.25 in

$$\sigma_{max,\, compression} = -\frac{(320\ lbs)}{\pi\left(4.5^2-4.25^2\right)} - \frac{(320\ ft\cdot lbs)\cdot 4.5}{\frac{\pi}{4}\left(4.5^4-4.25^4\right)} - \frac{(120lbs\cdot 120in)\cdot 4.5}{\frac{\pi}{4}\left(4.5^4-4.25^4\right)} - \frac{0.0549\cdot\pi\left(4.5^2-4.25^2\right)(120in)\cdot 32.2}{\pi\left(4.5^2-4.25^2\right)}$$

= **-1.505973093 > -40 ksi;** well within failure limit

**The density was converted to lbs/in^3*

B.3 Calculations for Shear Stress in Column (why it's neglected)

The shear stress for the cast iron 30 was determined using the formula:

$$\tau_{wind} = \frac{V_{wind}\cdot Q}{It} = \frac{120lbs\cdot\frac{2}{3}\left(r_o^3-r_i^3\right)}{\frac{\pi}{4}\left(r_o^4-r_i^4\right)\cdot 2r_o} = \mathbf{0.001939119528\ ksi.}$$

Since it was so small, we decided to exempt it in our stress analysis for simplicity.

B.4 Calculations for deflection

$M = n * w * 1ft * sin\theta$

Where n is the number of petals, w is the weight of each, and theta is the angle it makes with the x axis

M_{90° = 3840 lbm*in

$\delta_{max} = \frac{ML^2}{2EI}$

$\delta_{max} = \frac{PL^3}{3EI}$

Gray Cast Iron 30: r_0 = 4.0 in, r_i = 3.75 in

$\delta_{max} = \frac{3840*120^2}{2*15000*45.74} = 0.0403\ in$

+ wind

$\delta_{max} = \frac{120*120^3}{3*15000*45.74} = 0.101$ in

= 0.141in

Gray Cast Iron 60: r_0 = 3.75 in, r_i = 3.5 in

$\delta_{max} = \frac{3840*120^2}{2*20000*37.46} = 0.0369\ in$

+ wind

$\delta_{max} = \frac{120*120^3}{3*20000*37.46} = 0.0922$ in

= 0.129in

Stainless Steel 304: r_0 = 3.25 in, r_i = 3.0 in

$\delta_{max} = \frac{3840*120^2}{2*27500*24.01} = 0.0419\ in$

+ wind

$\delta_{max} = \frac{120*120^3}{3*27500*24.01} = 0.105$ in

= 0.147in

Aluminum Alloy: r_0 = 4.5 in, r_i = 4.25 in

$\delta_{max} = \frac{3840*120^2}{2*10150*65.82} = 0.0413\ in$

+ wind

$\delta_{max} = \frac{120*120^3}{3*10150*85.82} = 0.103$ in

= 0.145in

*All below the max deflection that would give us a FOS of 2 (0.15in)

B.5 Calculations for buckling

$W_c = 7.84\frac{EI}{L^2}$

Gray Cast Iron 30: r_0 = 4.0 in, r_i = 3.75 in

$W_c = 7.84\frac{15000*45.7}{120^2}$

= 373217 lb

Gray Cast Iron 60: r_0 = 4.0 in, r_i = 3.75 in

$W_c = 7.84\frac{20000*37.46}{120^2}$

= 407898 lb

Stainless Steel 304: r_0 = 3.25 in, r_i = 3.0 in

$W_c = 7.84\frac{27500*24.01}{120^2}$

= 359483 lb

Aluminum Alloy: r_0 = 4.5 in, r_i = 4.25 in

$W_c = 7.84\frac{10150*65.82}{120^2}$

= 363729 lb

B.6 Truss Calculations

Use superposition to derive an expression for displacement of the cantilever beam.

Total deflection

| |

Weight of the bar

$$v = \frac{-PL^2}{48EI}(6x - L) \quad L/2 \le x \le L$$

+

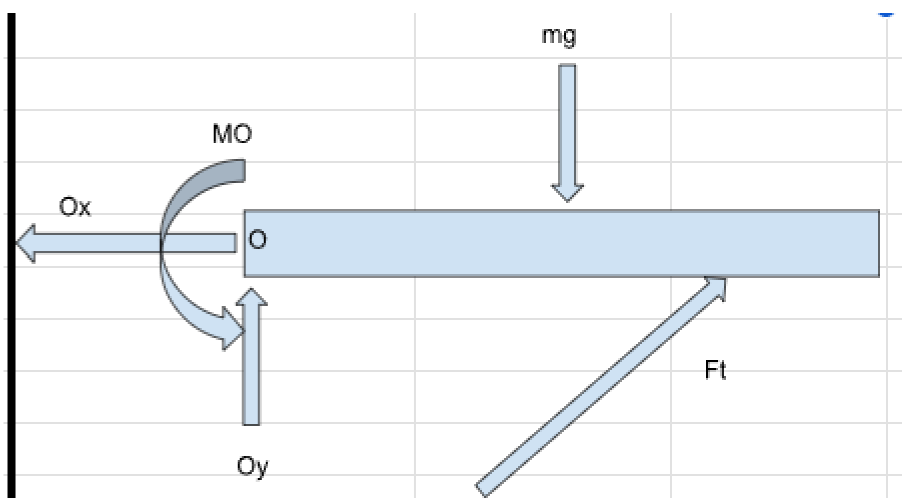
Force of the truss acting upward

$$y = \frac{Px^2}{6EI}(3a - x) \quad \text{for} \quad 0 < x < a$$

L = 60 in, x = 36in, a = 36in

Find highest force on truss in all three positions:

Ideal position:



Knowns: mg = 40lbs, EI = 145000 kip/in^2

Unknowns: Ox, Oy, Mo, Ft

Compatibility: At x = 36 ; v = 0 (where the truss connects)

$\Sigma Fx = 0 = Ox + Ft\,cos(18.434)$

$\Sigma Fy = 0 = Oy + Ft\,sin(18.434) - mg$

$\Sigma Mo = 0 = Mo + Ft\,sin(18.434) \cdot 36in - mg \cdot 30in$

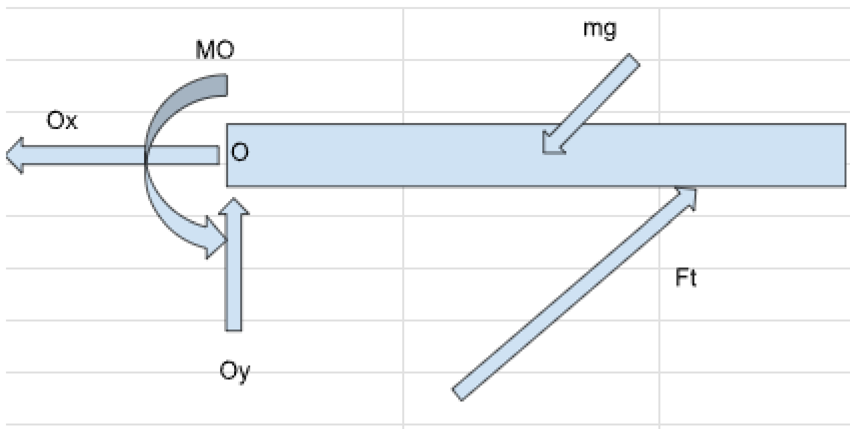
$$v = \frac{1}{EI}(\frac{mg\cdot(60in)^2}{48}(6\cdot 36in - 60in) - \frac{Ftsin(18.434)\cdot(36in)^2}{6}(3\cdot 36in - 36in))$$

Find Ft with v = 0

$$0 = (\frac{(40)\cdot(60in)^2}{48}(6\cdot 36in - 60in) - \frac{Ftsin(18.434)\cdot(36in)^2}{6}(3\cdot 36in - 36in))$$

$$Ft = \frac{\frac{(40)\cdot(60in)^2}{48}(6\cdot 36in - 60in)}{\frac{sin(18.434)\cdot(36in)^2}{6}(3\cdot 36in - 36in)} = 95.161lbs$$

Top side of position 2:



Knowns: mg = 40lbs, EI = 145000 kip/in^2
Unknowns : Ox, Oy, Mo, Ft
Compatibility: At x = 36 ; v = 0 (where the truss connects)

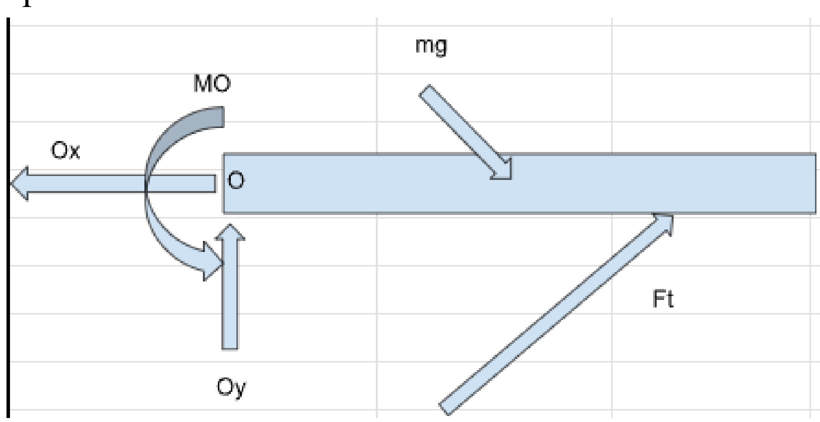
$$\Sigma Fx = 0 = Ox + Ft\,cos(18.434) - mgcos(45)$$

$$\Sigma Fy = 0 = Oy + Ft\,sin(18.434) - mgsin(45)$$

$$\Sigma Mo = 0 = Mo + Ft\,sin(18.434)\cdot 36in - mgsin(45)\cdot 30in$$

$$v = \frac{1}{EI}(\frac{mgsin(45)\cdot(60in)^2}{48}(6\cdot 36in - 60in) - \frac{Ftsin(18.434)\cdot(3in)6^2}{6}(3\cdot 36in - 36in))$$

Bottom side of position 2



Knowns: mg = 40lbs, EI = 145000 kip/in^2
Unknowns : Ox, Oy, Mo, Ft
Compatibility: At x = 36 ; v = 0 (where the truss connects)

$$\Sigma Fx \; = \; 0 \; = \; Ox \; + \; Ft\,cos(18.434) \; + \; mgcos(45)$$

$$\Sigma Fy \; = \; 0 \; = \; Oy \; + \; Ft\,sin(18.434) \; - \; mgsin(45)$$

$$\Sigma Mo \; = \; 0 \; = \; Mo \; + \; Ft\,sin(18.434) \cdot 36in \; - \; mgsin(45) \cdot 30in$$

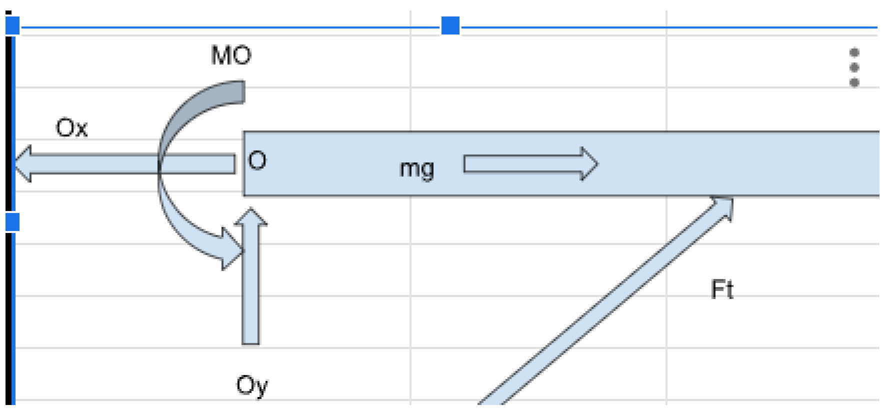
$$v \; = \; \frac{1}{EI}(\frac{mgsin(45)\cdot(6in)0^2}{48}(6 \cdot 36in - 60in) \; - \frac{Ftsin(18.434)\cdot(3in)6^2}{6}(3 \cdot 36in - 36in))$$

V equation same for both to find Ft
Find Ft with v = 0

$$0 \; = \; (\frac{(40sin(45))\cdot(60in)^2}{48}(6 \cdot 36in - 60in) \; - \frac{Ftsin(18.434)\cdot(3in)6^2}{6}(3 \cdot 36in - 36in))$$

$$Ft \; = \; \frac{\frac{(40sin(45))\cdot(60in)^2}{48}(6\cdot 36in - 60in)}{\frac{sin(18.434)\cdot(3in)6^2}{6}(3\cdot 36in - 36in)} \; = \; 67.289lbs$$

Vertical position



Knowns: mg = 40lbs, EI = 145000 kip/in^2
Unknowns: Ox, Oy, Mo, Ft
Compatibility: At x = 36in ; v = 0 (where the truss connects)

$$\Sigma Fx \; = \; 0 \; = \; Ox \; + \; Ft\,cos(18.434) \; + \; mg$$

$$\Sigma Fy \; = \; 0 \; = \; Oy \; + \; Ft\,sin(18.434)$$

$$\Sigma Mo \; = \; 0 \; = \; Mo \; + \; Ft\,sin(18.434) \cdot 36\,in$$

$$v \; = \; \frac{1}{EI}(- \frac{Ftsin(18.434)\cdot(3in)6^2}{6}(3 \cdot 36in - 36in))$$

When v = 0

$$0 \; = \; \frac{1}{EI}(- \frac{Ftsin(18.434)\cdot 36^2}{6}(3 \cdot 36 - 36))$$

Ft must be 0

Since position 3 has the most force on it it will fail first so we will test materials and design in that orientation

Ft = 95.161lbs

Calculate radius of the truss for the force applied with all 4 materials

$P_{cr} = \frac{\pi^2 EI}{L^2}$

$\sigma = \frac{P}{A}$

$A = \pi r^2$

$L = 3.162ft = 37.947in$

$Pcr = F * F.O.S$

$Pcr = \frac{\pi \cdot (EI)}{(L)^2}$

$I = \frac{\pi r^4}{4}$

Brittle:

Gray cast iron 30

EI = 15000 ksi

$I = \frac{\pi r^4}{4}$

σult = 100 ksi

$\sigma allow$ = 50 ksi

$\sigma allow = F/\pi r^2$

$r = \sqrt{\frac{95.161 lbs}{50000\ lbs/in^2 \cdot \pi}} = 0.0246in$

$Pcr = F * F.O.S = 190.322lbs$

$Pcr = \frac{\pi \cdot (EI)}{(L)^2}$

$Pcr = \frac{\pi \cdot (15000000\ lbs/in^2 \cdot \frac{\pi r^4}{4})}{(37.947in)^2}$

$r = \sqrt[4]{\frac{4 \cdot (37.947in)^2 \cdot 190.322lbs}{\pi^3 \cdot 15000000\ lbs/in^2}} = 0.220in$

Gray cast iron 60

EI = 20000 ksi

σult = 180 ksi

$\sigma allow$ = 90 ksi

$\sigma allow = F/\pi r^2$

$r = \sqrt{\frac{95.161 lbs}{90000\ lbs/in^2 \cdot \pi}} = 0.0183in$

$Pcr = F * F.O.S = 190.322lbs$

$Pcr = \frac{\pi \cdot (EI)}{(L)^2}$

$Pcr = \frac{\pi \cdot (20000000\ lbs/in^2 \cdot \frac{\pi r^4}{4})}{(37.947in)^2}$

$r = \sqrt[4]{\frac{4 \cdot (37.947in)^2 \cdot 190.322lbs}{\pi^3 \cdot 20000000\ lbs/in^2}} = 0.205in$

Ductile:

<u>Stainless steel 304</u>

EI = 27500 ksi

$\sigma yield$ = 40 ksi

$\sigma allow$ = 20 ksi

$\sigma allow\ = F/\pi r^2$

$r\ =\ \sqrt{\frac{95.161 lbs}{20000\ lbs/in^2\cdot\pi}} = 0.0389in$

$Pcr\ = F\ *\ F.O.S =\ 190.322lbs$

$Pcr\ = \frac{\pi\cdot(EI)}{(L)^2}$

$Pcr\ = \frac{\pi\cdot(\ 27500000\ lbs/in^2\cdot\frac{\pi r^4}{4})}{(37.947in)^2}$

$r\ =\ \sqrt[4]{\frac{4\cdot(37.947in)^2\cdot 190.322lbs}{\pi^3\cdot 27500000\ lbs/in^2}} = 0.189in$

<u>Aluminum Alloy</u>

EI = 10150 ksi

$\sigma yield$ = 43ksi

$\sigma allow$ = 21.5ksi

$\sigma allow\ = F/\pi r^2$

$r\ =\ \sqrt{\frac{95.161 lbs}{21500\ lbs/in^2\cdot\pi}} = 0.375in$

$Pcr\ = F\ *\ F.O.S =\ 190.322lbs$

$Pcr\ = \frac{\pi\cdot(EI)}{(L)^2}$

$Pcr\ = \frac{\pi\cdot(\ 10150000\ lbs/in^2\cdot\frac{\pi r^4}{4})}{(37.947in)^2}$

$r\ =\ \sqrt[4]{\frac{4\cdot(37.947in)^2\cdot 190.322lbs}{\pi^3\cdot 10150000\ lbs/in^2}} = 0.243in$